Collapse-revivals and population trapping in the m-photon mazer

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We present a study of collapse-revival patterns that appear in the changes of atomic populations induced by the interaction of ultracold two-level atoms with electromagnetic cavities in resonance with an m-photon transition of the atoms (m-photon mazer). In particular, sech^2 and gaussian cavity mode profiles are considered and differences in the collapse-revival patterns are reported. The quantum theory of the m-photon mazer is written in the framework of the dressed-state coordinate formalism. Simple expressions for the atomic populations, the cavity photon statistics, and the reflection and transmission probabilities are given for any initial state of the atom-field system. Evidence for the population trapping phenomenon which suppress the collapse-revivals in the m-photon mazer is given.

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I. INTRODUCTION

Laser cooling of atoms is a rapidly developing field in quantum optics. Cold and ultracold atoms introduce new regimes in atomic physics often not considered in the past. Recently, Scully *et al.* [1] have shown that a new kind of induced emission occurs when a micromaser is pumped by ultracold atoms, requiring a quantum-mechanical treatment of the center-of-mass motion. They called this particular process mazer action to insist on the quantized z-motion feature of the induced emission.

The detailed quantum theory of the mazer has been presented in a series of three papers by Scully and coworkers [2–4]. They showed that the induced emission probability is strongly dependent on the cavity mode profile. Analytical calculations were presented for the mesa and the sech^2 mode profiles. For sinusoidal modes, WKB solutions were detailed.

Retamal *et al.* [5] showed that we must go beyond the WKB solutions for the sinusoidal mode case when we consider strictly the ultracold regime. Remarkably, they showed that the resonances in the emission probability are not completely smeared out for actual interaction and cavity parameters. In a recent work [6], we proposed a numerical method for calculating efficiently the induced emission probability for arbitrary cavity field modes. In particular, the gaussian potential was considered, thinking in open cavities in the microwave or optical field regime. Differences with respect to the sech^2 mode case were found. Calculations for sinusoidal potentials were also performed and divergences with WKB results were reported, confirming results given in [5].

Zhang *et al.* [7] extended the concept of the mazer to the two-photon process by proposing the idea of the two-photon mazer. Their work was focused on the study of its induced emission probability in the special case of the mesa mode function. Under the condition of an initial coherent field state, they showed that this probability exhibits with respect to the interaction length the collapse-revivals phenomenon, which have different features in different regimes. They are similar to those in the two-photon Jaynes-Cummings model only in the thermal-atom regime.

The collapse-revivals of the atomic excitation in the framework of the Jaynes-Cummings model was predicted in the early 1980s by Eberly and co-workers [8–10]. Fleischhauer and Schleich [11] showed later that the shape of each revival is a direct reflection of the shape of the initial photon-number distribution P_n, assuming that the atom is prepared completely in the upper state or in the lower state and that the distribution P_n is sufficiently smooth. It was also noticed that, under some special conditions of the initial atom-field state, the revivals can be largely and even completely suppressed [12–14]. This phenomenon was denominated "population trapping" to refer, as noted by Yoo and Eberly [15], to a persistent probability of finding the atom in a given level in spite of the existence of both the radiation field and allowed transitions to other levels. The initial atom-field states giving rise to this phenomenon were called "trapping states" in [16]. Let us mention that this denomination is actually used in various physical contexts whenever a degree of freedom is found unaltered in spite of the existence of an interaction able to change its value. For instance trapping states in the context of the micromaser theory have been predicted and very recently measured by Filipowicz *et al.* [17] and Weidinger *et al.* [18] respectively. Nevertheless, these trapping states do not relate with the suppression of the collapse-revivals we are dealing here.

An elegant explanation of the population trapping phenomenon has been proposed just very recently by Jonathan *et al.* [19], who noticed that the key to understand the collapse-revival patterns under very general

conditions is to consider the joint initial properties of the atom-field system, even if this one is completely disentangled before the interaction. By defining an appropriate coordinate system, the dressed-state coordinates, they were able to yield simple analytical expressions for the atomic populations which exhibit the conditions needed for population trapping.

At the present time, no work has been devoted to know whether the revivals predicted by Zhang *et al.* [7] for the two-photon mazer may also be suppressed by use of an appropriate initial state of the atom-field system. An answer to this question is given at the end of this paper. To be not restricted to the case of the two-photon mazer, our analysis is generalized to the arbitrary m-photon mazer system, although the construction of real multiphoton cavities results in a formidable experimental task.

In Sec. II, we write the quantum theory of the m-photon mazer by use of the dressed-state coordinate formalism as it was very efficient in the description of the population trapping phenomenon in the Jaynes-Cummings model [19]. General expressions are derived for the atomic populations and the cavity photon distribution after the interaction of the atom with the cavity. The theory is written for any initial pure state of the atom-field system (entangled or not). We consider zero temperature and no dissipation in the high-Q cavity. In Sec. III, results of Zhang *et al.* [7] are extended to the 1, 2 and 3-photon mazer systems and to various cavity mode profiles (mesa, sech^2, and gaussian ones). Collapse-revival patterns are described for atoms prepared completely in the upper state or in the lower state. Sec. IV is devoted to the study of the m-photon mazer trapping states which suppress the collapse-revivals. A brief summary of our results is given in Sec. V.

II. THE MODEL

A. The Hamiltonian

We consider a two-level atom moving along the z-direction in the way to a cavity of length L. The atom is coupled resonantly with an m-photon transition to a single mode of the quantized field present in the cavity. The atom-field interaction is modulated by the cavity field mode function. The atomic center-of-mass motion is described quantum mechanically and the rotating-wave approximation is made. In the interaction picture, the Hamiltonian describing the system is

$$H = \frac{p^2}{2M} + \hbar g\, u(z)(a^{\dagger m}\sigma + a^m \sigma^\dagger), \tag{1}$$

where p is the atomic center-of-mass momentum along the z-axis, M is the atomic mass, $\sigma = |b\rangle\langle a|$ ($|a\rangle$ and $|b\rangle$ are respectively the upper and lower levels of the m-photon transition), a and $a^\dagger$ are respectively the annihilation and creation operators of the cavity radiation field, g is the atom-field coupling strength (half the Rabi frequency) and $u(z)$ is the cavity field mode.

B. The wavefunctions

In the z-representation and in the dressed-state basis

$$\begin{cases} |b,0\rangle, \ldots, |b,m-1\rangle, \\ |\pm,n\rangle = \frac{1}{\sqrt{2}}\left(|a,n\rangle \pm |b,n+m\rangle\right), \end{cases} \tag{2}$$

$|n\rangle$ being the photon-number states, the problem reduces to the scattering of the atom upon the potentials $V_n^\pm(z) = \pm\hbar g\sqrt{(n+1)\ldots(n+m)}\,u(z)$. Indeed, the set of wavefunction components

$$\psi_n^\pm(z,t) = \langle z,\pm,n|\psi(t)\rangle, \tag{3}$$

where $|\psi(t)\rangle$ is the atom-field state satisfy the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi_n^\pm(z,t) = \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + V_n^\pm(z)\right)\psi_n^\pm(z,t). \tag{4}$$

The general solution of (4) is

$$\psi_n^\pm(z,t) = \int dk\, \phi_n^\pm(k) e^{-i\frac{\hbar k^2}{2M}t}\varphi_n^\pm(k,z), \tag{5}$$

where $\varphi_n^\pm(k,z)$ is solution of the time-independent Schrödinger equation

$$\left(\frac{\partial^2}{\partial z^2} + k^2 \mp \kappa_n^2 u(z)\right)\varphi_n^\pm(k,z) = 0, \tag{6}$$

with

$$\kappa_n = \kappa\sqrt[4]{(n+1)\ldots(n+m)} \tag{7}$$

and

$$\kappa = \sqrt{2Mg/\hbar}\,. \tag{8}$$

The wavefunction components ($n = 1,\ldots,m$)

$$\psi_{-n}(z,t) = \langle z,b,m-n|\psi(t)\rangle \tag{9}$$

satisfy a Schrödinger equation characterized with a null potential and are therefore not affected by the interaction of the atom with the cavity. The atom in the lower state cannot obviously interact with the cavity field that contains less than m photons. The components (9) describe a free particle problem.

We assume that, initially, the atomic center-of-mass motion is not correlated to the other degrees of freedom. We describe it by the wave packet

$$\chi(z) \equiv \langle z|\chi\rangle = \int dk A(k) e^{ikz}\theta(-z), \tag{10}$$

where $\theta(z)$ is the Heaviside step function (indicating that the atoms are incident from the left of the cavity). No restrictions are made for the initial conditions of the atomic internal state and the cavity field state, except that pure states are only considered. By use of an expansion over the dressed-state basis (2), we may write

$$|\psi(0)\rangle = |\chi\rangle \otimes \left(\sum_{n=1}^{m} w_{-n} e^{i\chi_{-n}} |b, m-n\rangle + \sum_{n=0}^{\infty} w_n e^{i\chi_n} |\beta_n\rangle \right), \tag{11}$$

with

$$|\beta_n\rangle = \cos\left(\frac{\theta_n}{2}\right) |+, n\rangle + e^{-i\phi_n} \sin\left(\frac{\theta_n}{2}\right) |-, n\rangle. \tag{12}$$

The parameters $w_n \in [0,1]$, $\theta_n \in [0,\pi]$ and χ_n, $\phi_n \in [0, 2\pi]$ are called dressed-state coordinates [19]. The normalisation condition is

$$\sum_{n=-m}^{\infty} w_n^2 = 1 \tag{13}$$

and the phase factor χ_{-m} may be set to 0 without loss of generality.

We consider therefore

$$\begin{cases} \psi_{-n}(z,0) = c_{-n}\chi(z), \\ \psi_n^{\pm}(z,0) = c_n^{\pm}\chi(z), \end{cases} \tag{14}$$

with

$$\begin{cases} c_{-n} = w_{-n} e^{i\chi_{-n}}, \\ c_n^{+} = w_n e^{i\chi_n} \cos(\theta_n/2), \\ c_n^{-} = w_n e^{i(\chi_n - \phi_n)} \sin(\theta_n/2). \end{cases} \tag{15}$$

Inserting Eqs. (2) and (10) into Eq. (11), we get

$$\begin{aligned} |\psi(0)\rangle = &\int dz \int dk\, A(k) \times \\ &\left(\sum_{n=0}^{\infty} \Big[S_{a,n} e^{ikz} \theta(-z) |z, a, n\rangle \right. \\ &+ S_{b,n+m} e^{ikz} \theta(-z) |z, b, n+m\rangle \Big] \\ &\left. + \sum_{n=1}^{m} w_{-n} e^{i\chi_{-n}} e^{ikz} \theta(-z) |z, b, m-n\rangle \right), \end{aligned} \tag{16}$$

with

$$\begin{pmatrix} S_{a,n} \\ S_{b,n+m} \end{pmatrix} = \tilde{A}_n \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{17}$$

and

$$\tilde{A}_n = \frac{w_n e^{i\chi_n}}{\sqrt{2}} \begin{pmatrix} \cos(\theta_n/2) & e^{-i\phi_n} \sin(\theta_n/2), \\ \cos(\theta_n/2) & -e^{-i\phi_n} \sin(\theta_n/2), \end{pmatrix} \tag{18}$$

After the atom has left the interaction region, the wavefunctions $\varphi_n^{\pm}(k,z)$ can be written as

$$\varphi_n^{\pm}(k,z) = \begin{cases} r_n^{\pm}(k) e^{-ikz} & (z<0) \\ t_n^{\pm}(k) e^{ik(z-L)} & (z>L) \end{cases}, \tag{19}$$

where $r_n^{\pm}(k)$ and $t_n^{\pm}(k)$ are respectively the reflection and transmission coefficient associated with the scattering of the particle of momentum $\hbar k$ upon the potential $V_n^{\pm}(z)$ (Eq. 6). The initial state components $\psi_n^{\pm}(z,0)$ have evolved into

$$\begin{aligned} \psi_n^{\pm}(z,t) = c_n^{\pm} \int dk\, A(k) e^{-i\frac{\hbar k^2}{2M}t} \big[r_n^{\pm}(k) e^{-ikz} \theta(-z) \\ + t_n^{\pm}(k) e^{ik(z-L)} \theta(z-L) \big] \end{aligned} \tag{20}$$

whereas the free particle wavefunction components $\psi_{-n}(z,0)$ become

$$\psi_{-n}(z,t) = c_{-n} \int dk\, A(k) e^{-i\frac{\hbar k^2}{2M}t} e^{ik(z-L)} \theta(z-L). \tag{21}$$

We thus obtain

$$\begin{aligned} |\psi(t)\rangle = &\int dz \int dk\, A(k) e^{-i\frac{\hbar k^2}{2M}t} \times \\ &\left(\sum_{n=0}^{\infty} \Big[R_{a,n}(k) e^{-ikz} \theta(-z) |z, a, n\rangle \right. \\ &+ T_{a,n}(k) e^{ik(z-L)} \theta(z-L) |z, a, n\rangle \\ &+ R_{b,n+m}(k) e^{-ikz} \theta(-z) |z, b, n+m\rangle \\ &+ T_{b,n+m}(k) e^{ik(z-L)} \theta(z-L) |z, b, n+m\rangle \Big] \\ &\left. + \sum_{n=1}^{m} w_{-n} e^{i\chi_{-n}} e^{ik(z-L)} \theta(z-L) |z, b, m-n\rangle \right), \end{aligned} \tag{22}$$

in which

$$\begin{pmatrix} R_{a,n}(k) \\ R_{b,n+m}(k) \end{pmatrix} = \tilde{A}_n \begin{pmatrix} r_n^{+}(k) \\ r_n^{-}(k) \end{pmatrix}, \tag{23a}$$

$$\begin{pmatrix} T_{a,n}(k) \\ T_{b,n+m}(k) \end{pmatrix} = \tilde{A}_n \begin{pmatrix} t_n^{+}(k) \\ t_n^{-}(k) \end{pmatrix}. \tag{23b}$$

If initially the electromagnetic field is in the state $|n\rangle$ and the atom is in the excited state $|a\rangle$, the only non-zero dressed-state coordinates are $w_n = 1$ and $\theta_n = \pi/2$. We get therefore

$$\tilde{A}_n = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{24}$$

and Eqs. (23) lead to the same results given by Meyer *et al.* [2] who considered in detail this case for the one-photon mazer.

C. Atomic populations

The reduced density matrix $\sigma(t)$ for the atomic internal degree of freedom is given by the trace over the radiation and the atomic external variables of the atom-field density matrix, that is its elements $i, j = a, b$ are

$$\sigma_{ij}(t) = \sum_n \int dz \langle z, i, n|\psi(t)\rangle\langle\psi(t)|z, j, n\rangle. \quad (25)$$

The atomic populations σ_{ii} follows immediately from Eq. (25) :

$$\sigma_{ii}(t) = \sum_n \int dz |\langle z, i, n|\psi(t)\rangle|^2. \quad (26)$$

Inserting Eqs. (16) and (22) into Eq. (26) and using Eqs. (17) and (23), we get for an incident atom of momentum $\hbar k$:

$$\sigma_{aa}(0) = \frac{1}{2}\left[1 - \sum_{n=1}^{m} w_{-n}^2 + \sum_{n=0}^{\infty} w_n^2 \sin(\theta_n)\cos(\phi_n)\right], \quad (27)$$

$$\sigma_{aa}(t) = \frac{1}{2}\left[1 - \sum_{n=1}^{m} w_{-n}^2 + \sum_{n=0}^{\infty} w_n^2 \sin(\theta_n)\mathrm{Re}(e^{i\phi_n} K_n)\right], \quad (28)$$

where

$$K_n = r_n^+ r_n^{-*} + t_n^+ t_n^{-*}. \quad (29)$$

The change of the atomic population σ_{aa} induced by the interaction of the incident atom with the cavity radiation field is then given by

$$\delta\sigma_{aa} = \sigma_{aa}(t) - \sigma_{aa}(0), \quad (30)$$

with the time t chosen long after the interaction.

Thus we have

$$\delta\sigma_{aa} = \sum_{n=0}^{\infty} \Delta_n, \quad (31)$$

with

$$\Delta_n = \frac{w_n^2}{2}\sin(\theta_n)\left[\mathrm{Re}\left(e^{i\phi_n} K_n\right) - \cos(\phi_n)\right]. \quad (32)$$

As expected, the components $w_{-n}e^{i\chi_{-n}}$ of the initial state $|\psi(0)\rangle$ over the states $|b, n\rangle$ $(n < m)$ do not play any role in the dynamics of the system.

We have to emphasize that in Eq. (31) Δ_n *cannot* be interpreted strictly as the change in the σ_{aa} population induced by the interaction of the two-level atom with the cavity radiation field containing n photons. This is only true when the incident atom is prepared in the excited state. Indeed, if initially the internal atomic state is $c_a|a\rangle + c_b|b\rangle$ and the field state is $|n\rangle$ $(n \geq m)$, then the only non-zero dressed-state coordinates are $w_n = |c_a|$, $\chi_n = \arg(c_a)$, $\theta_n = \pi/2$, $w_{n-m} = |c_b|$, $\chi_{n-m} = \arg(c_b)$, $\theta_{n-m} = \pi/2$ and $\phi_{n-m} = \pi$. We thus have in that case

$$\begin{aligned}\delta\sigma_{aa} &= \Delta_n + \Delta_{n-m}, \\ &= \Delta_n \quad \text{iff} \quad c_b = 0.\end{aligned} \quad (33)$$

D. Photon statistics

The reduced density matrix $\rho(t)$ for the cavity radiation field is given by the trace over the internal and external atomic degrees of freedom of the atom-field density matrix, that is its elements n, n' are

$$\rho_{nn'}(t) = \sum_{i=a,b} \int dz \langle z, i, n|\psi(t)\rangle\langle\psi(t)|z, i, n'\rangle. \quad (34)$$

The photon distribution $P_n = \rho_{nn}$ follows immediately from Eq. (34) :

$$P_n(t) = \sum_{i=a,b} \int dz |\langle z, i, n|\psi(t)\rangle|^2. \quad (35)$$

The change δP_n in the cavity photon distribution induced by the interaction of the cavity electromagnetic field with the incident atom is then given by

$$\delta P_n = P_n(t) - P_n(0). \quad (36)$$

Inserting Eqs. (16) and (22) into Eq. (35) and using Eqs. (17) and (23), we get for an incident atom of momentum $\hbar k$:

$$\delta P_n = \begin{cases} \Delta_n - \Delta_{n-m} & (n \geq m), \\ \Delta_n & (n < m). \end{cases} \quad (37)$$

We see that if the initial state is $|a, n\rangle$ we have

$$\delta\sigma_{aa} + \delta P_{n+m} = 0, \quad (38)$$

which gives an intuitive population conservation condition.

E. Reflection and transmission probabilities

The reflection and transmission probabilities of the incident atom upon the cavity are respectively given by

$$R = \sum_{i=a,b}\sum_n \int_{-\infty}^{0} dz |\langle z, i, n|\psi(t)\rangle|^2, \quad (39a)$$

$$T = \sum_{i=a,b}\sum_n \int_{L}^{\infty} dz |\langle z, i, n|\psi(t)\rangle|^2. \quad (39b)$$

Inserting Eq. (22) into Eqs. (39), we get for an incident atom of momentum $\hbar k$:

$$R = \sum_{n=0}^{\infty} w_n^2 \Big(\cos^2(\theta_n/2)|r_n^+|^2 + \sin^2(\theta_n/2)|r_n^-|^2\Big), \quad (40a)$$

$$T = \sum_{n=0}^{\infty} w_n^2 \Big(\cos^2(\theta_n/2)|t_n^+|^2 + \sin^2(\theta_n/2)|t_n^-|^2\Big) + \sum_{n=1}^{m} w_{-n}^2. \quad (40b)$$

One verifies immediately that the results of Meyer *et al.* [2] about the reflection and transmission probabilities are well recovered by Eqs. (40) when their initial conditions are considered. Indeed, when the atom-field system is initially in the state $|a,n\rangle$, Eqs. (40) become

$$R = \frac{1}{2}\big(|r_n^+|^2 + |r_n^-|^2\big), \tag{41a}$$

$$T = \frac{1}{2}\big(|t_n^+|^2 + |t_n^-|^2\big). \tag{41b}$$

We get the same results if the atom-field system is initially in the state $|b,n\rangle$ with $n \geq m$, except that n must be replaced by $n-m$ in Eqs. (41). In the case $n < m$, we have obviously $T = 1$.

F. Final remarks

All the results here above (about the atomic populations, the photon statistics, and the reflection and transmission probabilities) may be very easily generalized for any momentum wavefunction $A(k)$ of the initial wave packet. The various expressions must simply be weighted by $|A(k)|^2$ and integrated over k. For instance, Eq. (31) becomes

$$\delta\sigma_{aa} = \int dk\, |A(k)|^2 \sum_{n=0}^{\infty} \Delta_n, \tag{42}$$

where Δ_n depends on k through the reflection and transmission coefficients, $r_n^\pm(k)$ and $t_n^\pm(k)$ respectively, in K_n (see Eq. (32)).

The expressions obtained for all these various physical quantities are very simple in the framework of the dressed-state formalism, even though they are very general. They take a form much more complicated when the usual coordinates of the atom-field system are used (the complex coefficients c_a, c_b and $c(n)$ of the atom-field states written as $(c_a|a\rangle + c_b|b\rangle) \otimes \sum_n c(n)|n\rangle$). Also entangled initial states may be considered by this formalism. The great advantage of the dressed-state coordinates was already pointed out by Jonathan *et al.* [19] who used them to express various physical quantities in the Jaynes-Cummings model.

III. COLLAPSE-REVIVALS

Expressions (31) and (37) show that the features of the changes in the atomic populations and in the photon distribution P_n with respect to the interaction length κL are directly related to the characteristics of Δ_n. This in turn depends on the atom-field initial state (through the dressed-state coordinates) and on the cavity field mode profile $u(z)$ which affect the reflection and transmission coefficients, $r_n^\pm$ and $t_n^\pm$ respectively, and thus K_n.

If K_n have a strong oscillatory behaviour with respect to κL, we may expect collapse-revivals in the population changes when several modes of the field are initially filled. In the following we will restrict ourselves to the description of the collapse-revivals when the atom is prepared completely in the upper or in the lower state, and the field is in the state $\sum_n c(n)|n\rangle$.

When the atom is initially in the upper state $|a\rangle$, $\sigma_{bb}(t) = 1 - \sigma_{aa}(t)$ represents the probability that a photon be emitted by the atom due to its interaction with the cavity. From Eq. (28), one gets that this induced emission probability is given by

$$P_{em} = \sum_{n=0}^{\infty} p(n) P_{em}(n), \tag{43}$$

with $p(n) = |c(n)|^2$ and

$$P_{em}(n) = \frac{1 - \mathrm{Re}(K_n)}{2}. \tag{44}$$

When the atom is initially in the lower state $|b\rangle$, $\sigma_{aa}(t)$ represents the probability that a cavity photon be absorbed by the atom. This absorption probability is identical to the induced emission probability P_{em}, except that $p(n)$ must be replaced by $p(n+m)$ in Eq. (43).

The induced emission probability P_{em} is studied hereafter for different cavity mode profiles: mesa, sech^2 and gaussian modes. Our description is restricted to the ultracold regime (incident atoms with a momentum $\hbar k$ such that $k/\kappa \ll 1$).

A. Mesa mode

In the special case where the cavity field mode profile is given by the mesa function

$$u(z) = \begin{cases} 1 & \text{for } 0 < z < L \\ 0 & \text{elsewhere} \end{cases} \tag{45}$$

the reflection and transmission coefficients $r_n^\pm(k)$ and $t_n^\pm(k)$ respectively may be calculated analytically. Their expression has been given for the one-photon mazer by Löffler *et al.* [3]. Same results are obtained for the m-photon mazer, except that the value of the parameter κ_n must be changed accordingly (see Eq. (7)). Inserting these results into Eq. (44), one gets when $\exp(\kappa_n L) \gg 1$ and $(\kappa_n/2k)^2 \exp(\kappa_n L)\sin(\kappa_n L) \gg 1$ that

$$P_{em}(n) = \frac{\frac{1}{2}\left[1 + \frac{1}{2}\sin(2\kappa_n L)\right]}{1 + (\kappa_n/2k)^2 \sin^2(\kappa_n L)}. \tag{46}$$

As pointed out by Löffler *et al.* [3], Eq. (46) is similar to the Airy function of the classical optics which gives the transmitted intensity in a Fabry-Perot interferometer. This by no means exhibits a strong oscillatory behaviour. Thus, we have no chance to obtain similar

collapse-revivals as those in the Jaynes-Cummings model when several mode of the field are initially filled. This is illustrated in Fig. 1 for the 1, 2 and 3-photon mazers where the cavity field is taken initially in a coherent state ($p(n) = e^{-\bar{n}}\frac{\bar{n}^n}{n!}$) with a mean photon number $\bar{n} = 10$. A chaotic behaviour in the curves $P_{em}(\kappa L)$ is clearly obtained for $k/\kappa = 0.1$.

B. Sech^2 mode

When the cavity field mode profile is given by the sech^2 function

$$u(z) = \mathrm{sech}^2(z/L) \tag{47}$$

the reflection and transmission coefficients may also be calculated analytically. Their expression has been given for the one-photon mazer by Löffler *et al.* [3]. Again, same results are obtained for the m-photon mazer, except that the value of the parameter κ_n must be changed accordingly. Hence, the curves $P_{em}(n)$ with respect to the interaction length $\kappa_n L$ are identical for the one-photon and the m-photon mazers. Two such curves have been presented by Löffler *et al.* [3] for $k/\kappa_n = 0.1$ and $k/\kappa_n = 0.01$. These curves present well resolved resonances that get smeared for large values of the interaction length. We have calculated for the 1, 2 and 3-photon mazers the induced emission probability in the case of a cavity field initially in a coherent state (with $\bar{n} = 10$). These results are presented on Fig. 2 for $k/\kappa = 0.1$. Evidence for collapse-revivals is shown on these figures. They are stronger in the case of the 3-photon mazer.

C. Gaussian mode

For a cavity field mode profile described by the gaussian function

$$u(z) = e^{-\frac{z^2}{2\sigma^2}} \tag{48}$$

the reflection and transmission coefficients can no more be calculated analytically.

We proposed recently [6] a numerical method for computing efficiently these coefficients and the induced emission probability $P_{em}(n)$. We compare on Fig. 3 the results obtained for this probability to those calculated in the case of the sech^2 mode profile. We have considered $k/\kappa_n = 0.1$ and interaction lengths $\kappa_n L$ varying between 0 and 20. The parameter σ in Eq. (48) was fixed to $\sqrt{2/\pi}L$ in order to adopt the same normalization factor for the two profiles (identical area under the modes). As we pointed out in [6], the resonances in the curves get smeared with increasing values of $\kappa_n L$ for both profiles. But this fact is not so marked in the case of the gaussian profile where the resonances still exist for longer interaction lengths. It is a result to be expected as the gaussian profile is growing more abruptly than the sech^2 one. Thus it is in some sense "closer" to the mesa mode, which exhibits resonances at infinity.

We have then calculated the induced emission probability P_{em} with respect to κL for a field initially in a coherent state (with $\bar{n} = 10$). The result is presented on Fig. 4 for the 1, 2 and 3-photon mazers. As the curves for the probability $P_{em}(n)$ are qualitatively similar in the cases of the gaussian and the sech^2 modes, it is not a surprising result that the collapse-revivals are also similar in both cases. Nevertheless, they are stronger for the gaussian potential because the oscillations in $P_{em}(n)$ are stronger too.

Squeezing the field inside the cavity has an effect on the collapse-revival patterns. We have considered initial photon distributions $p(n)$ inside the cavity corresponding to various squeezed coherent states $|\alpha, re^{i\theta}\rangle$, namely

$$p(n) = \frac{(\tanh r)^n}{2^n n! \cosh r} \left| H_n\left(\frac{\alpha e^{-i\theta/2}}{\sqrt{2\cosh r \sinh r}}\right)\right|^2 \times \exp\left[-|\alpha|^2 + \frac{1}{2}(e^{-i\theta}\alpha^2 + e^{i\theta}(\alpha^*)^2)\tanh r\right], \tag{49}$$

where $H_n(z)$ designates the n^{th} order Hermite polynomial. We noticed that squeeze parameters r of the order of 0.3 enhance significantly the collapse-revivals presented on Figs. 2 and 4 (keeping the same coherent parameter $|\alpha|^2 = 10$ and taking $\theta = 0$), while higher squeeze parameters tend to destroy them.

IV. POPULATION TRAPPING

When the atom-field initial state is such that $\sin(\theta_n) = 0$, we get from Eq. (32) $\Delta_n = 0$, whatever the value of K_n. In this case, we have

$$\delta\sigma_{aa} = \delta\sigma_{bb} = \delta P_n = 0, \tag{50}$$

indicating that the interaction of the atom with the cavity radiation field has no effect on the atomic populations σ_{ii} $(i = a, b)$ and on the cavity photon distribution P_n, whatever the cavity field mode function, whatever the cavity interaction length κL and whatever the atomic initial velocity. We conclude that the mazer give rise to the perfect population trapping phenomenon, when considering zero temperature and no dissipation in the high-Q cavity. This property holds for the ultracold, intermediate and thermal-atom regimes, as it is completely independent on the external atomic degree of freedom. For the same reason, it holds for any momentum wavefunction $A(k)$ of the initial wave packet.

The class of states verifying $\sin(\theta_n) = 0$, named *perfect trapping states*, are given by

$$|\gamma^\pm\rangle = \frac{\gamma^m|a\rangle \pm |b\rangle}{\sqrt{1+|\gamma|^{2m}}} \otimes \sqrt{1-|\gamma|^2}\sum_{n=0}^{\infty}\gamma^n|n\rangle, \tag{51}$$

where γ is a complex number with $|\gamma| < 1$.

Indeed, rewriting these states in terms of the dressed-state basis, we find

$$|\gamma^\pm\rangle = \sqrt{\frac{1-|\gamma|^2}{1+|\gamma|^{2m}}} \left(\sum_{n=0}^{\infty} \sqrt{2}\gamma^{n+m}|\pm, n\rangle \pm \sum_{n=0}^{m-1} \gamma^n |b, n\rangle \right). \tag{52}$$

For each n there is only a single dressed-state present in the sum of expression (52). Depending on whether it is $|+, n\rangle$ or $|-, n\rangle$, we have respectively $\sin(\theta_n/2) = 0$ or $\cos(\theta_n/2) = 0$, and so $\sin(\theta_n) = 0$ in any case.

This give rise to another very interesting feature of the perfect trapping states. The reflection and transmission probabilities (40) become

$$R = \sum_{n=0}^{\infty} w_n^2 |r_n^\pm|^2, \tag{53a}$$

$$T = \sum_{n=0}^{\infty} w_n^2 |t_n^\pm|^2 + \sum_{n=1}^{m} w_{-n}^2, \tag{53b}$$

with

$$w_n = \sqrt{\frac{1-|\gamma|^2}{1+|\gamma|^{2m}}} \sqrt{2}|\gamma|^{n+m}, \tag{54a}$$

$$w_{-n} = \sqrt{\frac{1-|\gamma|^2}{1+|\gamma|^{2m}}} |\gamma|^{m-n}. \tag{54b}$$

The particle moving along the z-axis is only sensitive to either a superposition of the potentials $V_n^+(z)$ or a superposition of $V_n^-(z)$, but never to both. In principle, it would be possible to imagine an experimental set-up where the particles would encounter only an effective potential well, instead of an effective potential hill.

It is important to emphasize that the perfect trapping states do not make the cavity transparent to the incident atoms, because the reflection coefficient R is not nullified.

V. SUMMARY

In this paper, we have studied collapse-revival patterns that appear in the changes of the atomic populations induced by the interaction of ultracold two-level atoms with electromagnetic cavities of various interaction lengths that are in resonance with an m-photon transition of the atoms. In particular, the sech^2 and gaussian cavity mode profiles have been considered and differences in the collapse-revival patterns are reported. They are stronger in the case of the gaussian potential. With the aim of such studies in view, we have written the quantum theory of the m-photon mazer by use of the dressed-state coordinate formalism. Simple expressions for the atomic populations, the cavity photon statistics, and the reflection and transmission probabilities have been given for any initial pure state of the atom-field system. The evidence for the population trapping phenomenon has then been very easily given. The trapping states written in Sec. IV have the property to leave, after the atom-field interaction, the cavity field and the internal atomic degrees of freedom at their initial value, independently of the cavity field mode, the cavity interaction length, and the initial atomic velocity.

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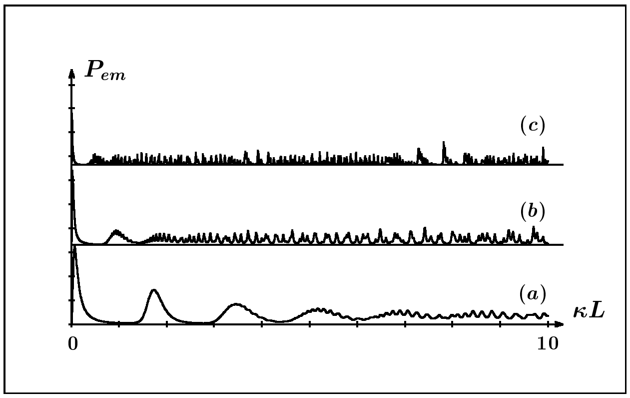

FIG. 1. The induced emission probability P_{em} as a function of the interaction length κL for the mesa mode profile and a cavity field initially in a coherent state ($\bar{n} = 10$, $k/\kappa = 0.1$). (a) 1-photon mazer, (b) 2-photon mazer, (c) 3-photon mazer. The y-scale is between 0 and 0.5 for each curve.

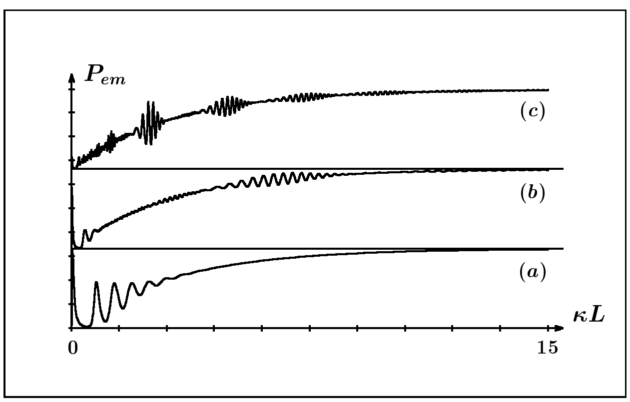

FIG. 2. The induced emission probability P_{em} as a function of the interaction length κL for the sech^2 mode profile and a cavity field initially in a coherent state ($\bar{n} = 10$, $k/\kappa = 0.1$). (a) 1-photon mazer, (b) 2-photon mazer, (c) 3-photon mazer. The y-scale is between 0 and 0.5 for each curve.

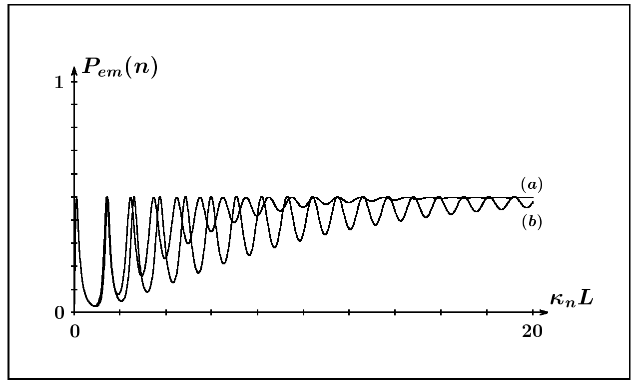

FIG. 3. The induced emission probability $P_{em}(n)$ as a function of the interaction length $\kappa_n L$ for the sech^2 mode profile (a) and the gaussian profile (b) ($k/\kappa_n = 0.1$).

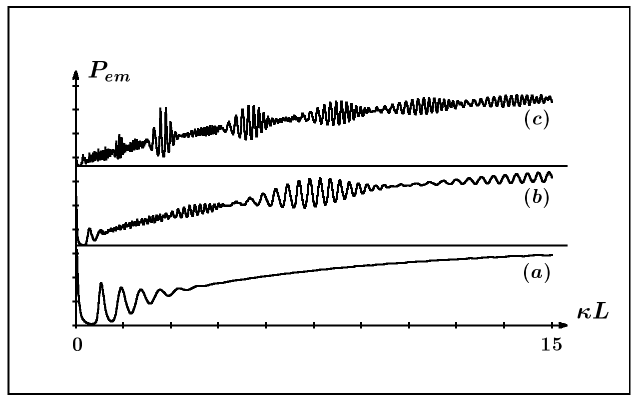

FIG. 4. The induced emission probability P_{em} as a function of the interaction length κL for the gaussian mode profile and a cavity field initially in a coherent state ($\bar{n} = 10$, $k/\kappa = 0.1$). (a) 1-photon mazer, (b) 2-photon mazer, (c) 3-photon mazer. The y-scale is between 0 and 0.5 for each curve.